RADIATION OF PHOTONS IN PROCESS OF CHARGE PARTICLE VOLUME REFLECTION IN BENT SINGLE CRYSTAL

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Abstract

New type of radiation in crystals is predicted and investigated in computer simulation. It is shown that process of volume reflection of electrons and positrons in bent crystals is accomplished with high-power radiation of photons. Volume reflection radiation has intensity comparable with known channeling radiation, but it is less sensitive to entrance angle and sign of charge of a particle. Simulated spectra of radiation power are presented for 10 GeV and 200GeV particles.

1 Introduction

Volume reflection of charged particles in bent single crystals was predicted in Ref. [1] as result of Monte Carlo simulations. Recently this process was observed experimentally in extracted proton beams [2]. At volume reflection the proton crosses a number of crystallographic planes. Due to this fact its motion has a complicated oscillating character. For this reason one can expect valuable radiation losses of energy for light particles (such as electrons and positrons).

In this paper we investigate the radiation of energetic photons by electrons (positrons) moving in bent single crystals at conditions of volume reflection effect. Our consideration based on the equations derived from quasi-classical operator method (see Ref.[3]). Within this method the probabilities of QED processes may be expressed by way of classical trajectories of charged particles in electrical fields.

Taking into account the above mentioned we start from consideration of motion of charged particles in bent single crystals and then we calculate the differential energy losses of radiation process. It should be noted that the analytical description of volume reflection one can find in paper [4].

2 Motion in bent single crystals

One can describe the motion of ultrarelativistic particles in bent single crystals with the help of the following equations[4] :

$$E_0 \beta^2 v_r^2/(2c^2) + U(r) + E_0 \beta^2 (R - r)/r = E = const, \quad (1)$$

$$dy/dt = v_y = const, \quad (2)$$

$$v_z = r d\phi/dt \approx c(1 - \frac{1}{2\gamma^2} - \frac{(v_r^2 + v_y^2)}{2c^2}). \quad (3)$$

These equations take place for the cylindrical coordinate system (r, ϕ, y). Here v_r is the component of particle velocity in the radial direction, v_y is the component of the velocity

along y-axis and v_z is the tangential component of the velocity, R is the radius of bending of a single crystal, E_0 and γ are the particle energy and its Lorentz factor, E is the constant value of the radial energy, $U(r)$ is the one dimensional potential of a single crystal, c is the velocity of light and β is the ratio of the particle velocity to velocity of light. In this paper we consider the planar case when the scattering is due to the interaction of particles with the set of the crystallographic planes located normally to (r, ϕ)-plane. On the practice it means that $v_y/c \gg \theta_{ac}$ but $v_y/c \ll 1$ for ultrarelativistic particles, where θ_{ac} is the critical angle of axial channeling.

Eq.(1) one can transform in the following form:

$$E_0 \beta^2 v_x^2/(2c^2) + U(x) + E_0 \beta^2 x/R = E, \tag{4}$$

where x is the local Cartesian coordinate which connected with the cylindrical coordinate r through the relation $x = R - r$ and $v_x = v_r$. We also changed r-value in the denominator of Eq.(1) on R. For real experimental situation it brings a negligible mistake (of the order of x/R). In Eq.(4) E-value have a sense of the transversal energy.

The considered here equations describe three dimensional motion of particles in a bent single crystal in the cylindrical coordinate system. Let us introduce the Cartesian coordinate system in which the xy-plane is coincident with the front edge of a single crystal. Now we can calculate x-component of the velocity in this system:

$$V_x = v_x \cos \phi + v_z \sin \phi \approx v_x + v_z \phi. \tag{5}$$

$$V_z = -v_x \sin \phi + v_z \cos \phi. \tag{6}$$

Fig.1 illustrates the geometry of particle volume reflection and different coordinate systems which will be used in our consideration.

With the help of Eq. (5) we can calculate the velocity V_x as a function of time. For specific calculations we select the energy of electron (positron) beam equal to 200 GeV and (110) plane of the silicon single crystal. The planar potential for this plane calculated on the basis of results of x-ray diffraction (see details in Ref.[4]). Fig. 2 illustrates the behavior of velocity V_x at symmetrical orientation of single crystal (when entering angle of particle is equal to exit angle). The radius of the curvature is equal to 10 meters and the thickness of the single crystal is approximately equal to 0.06 cm. One can see that particle performs an aperiodic oscillations in the transversal plane.

3   Radiation energy losses of particle

For calculations of radiation energy losses of particle we use the following relation (see [3]):

$$\frac{d\mathcal{E}}{dE_\gamma} = \frac{i\alpha m^2 c^4}{2\pi\varepsilon^2} \omega \int_{\mathbf{D}} \frac{dtd\tau}{\tau - 0} \left\{ 1 + \frac{\varepsilon^2 + \varepsilon'^2}{4c^2\varepsilon\varepsilon'} \gamma^2 [\Delta\mathbf{v}(t - \tau/2) - \Delta\mathbf{v}(t + \tau/2)]^2 \right\} \exp -iA_1, \tag{7}$$

$$A_1 = \frac{\omega\varepsilon\tau}{2\varepsilon'} \left[\frac{1}{\gamma^2} + \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} ds (\Delta\mathbf{v}(t + s)/c)^2 - \left(\frac{1}{\tau} \int_{-\tau/2}^{\tau/2} ds \Delta\mathbf{v}(t + s)/c \right)^2 \right], \tag{8}$$

where $\Delta\mathbf{v}(t, \mathbf{v}_0) = \mathbf{v}(t_1) - \mathbf{v}_0$ is the velocity variation as a function of time t_1, m and γ are the mass and Lorentz factor of particle, E_γ, ω are the energy and frequency of photon, ε is the particle energy, $\varepsilon' = \varepsilon - E_\gamma$. The time variables t_1 and t_2 (t_2 is time variable as t_1) connected with variables t and τ by equations: $t_1 = t - \tau/2$ and $t_2 = t + \tau/2$.

D is the domain of definition of integrand function. Eq. (7) has a common character and may be used for a solution of a numerous radiation problem. In particular with the help of this equation relations were obtained which describe the radiation at quasiperiodic motion, synchrotron like radiation and so on [3]. For these aims some simplifications of the integrand functions were made. Most difficult case for calculations is the case when the angle of deflection of moving particle on the formation length is compared with the characteristic radiation angle $(1/\gamma)$.

Consideration of the curves in Fig. 2 shows that the character of motion at volume reflection significantly differs from motion in a straight single crystal. The peculiarities are: aperiodicity of oscillations and deflection of particles (electrons) of order of characteristic radiation angle. The time for one oscillation changed from ≈ 50 fs (femptosecond) near critical point till ≈ 20 fs on the entrance (exit) of single crystal. The value of characteristic radiation angle (for condition in Fig.2) is equal to $\approx 2.5 \mu rad$. The variation of relative transversal particle velocity (corresponding changing of angle) is $\approx 10^{-5} - 210^{-6}$. Let us recall the following relation :

$$\omega = \frac{2\gamma^2 \omega_0}{1 + \rho/2},\qquad(9)$$

(where $\rho = 2\gamma^2 \overline{v_\perp^2}$ and $v_\perp$ is the transversal particle velocity) which defined a frequency of emitted photon by the frequency ω_0 of particle motion. In our case we should take the time T_0 of every oscillation and calculate frequency $\omega_0 = 2\pi/T_0$. Taking into account that parameter $\rho \sim 1$ we get estimation for photon energy $\sim \gamma^2 \omega_0$. Thus we see that more intensive range of photon spectrum (for condition as in Fig.2) corresponds to $15-35$ GeV. Another obvious peculiarity of the radiation at volume reflection is dependence the form of photon spectrum on the thickness of bent single crystal. Really with increasing of the thickness the number of periods also increases and their period is decreased. In this case one can expect increasing of number of high energy photons.

The calculation of radiation losses spectra with accordance Eq.(7) is serious computional problem, mainly due to oscillating character of the integrand function. However, we do not see clear possibility for a simplification of Eq.(7). We think that in this case the direct calculations with the use of Eq. (7) may be very useful for further understanding of process and allow one to get necessary simplifications.

For calculation of differential radiation energy losses we create program, which allow us to find photon distributions in accordance with Eq. (7). We made computations for radiation of 200 GeV electrons and positrons in thick silicon bent single crystal for conditions shown in Fig. 2. It should be noted that calculations are very time consuming. This time depends also on the accuracy of calculations. Our experience shows that for the same accuracy the calculations at higher energy require more time.

One can expect that the integration over τ in Eq. (7) may be performed in limits of time which necessary for formation of radiation. Partially, it is so. Really, main contribution in losses come from this range. However, there are oscillations of integrand function at large enough values τ which can change result on 5-15 %. If do not take this into account the integral may be calculated fast but accuracy will not high. Nevertheless, it may be possible way for simplification of Eq. (7).

Fig. 3 illustrates the result of calculation of differential radiation energy losses for 200 GeV electrons. According to our estimation the accuracy of this calculation is 1-2% for photon energies less then 45 GeV, and about 5% from 45 GeV till 85. As expected the main losses located in the range 15-40 GeV.

The differential energy losses of positrons presented in Fig. 3 show that the form of

photon spectrum is similar (in comparison with radiation of electrons) but intensity in range less than 45 Gev for positrons is approximately larger on $\approx 20\%$. We estimate accuracy of calculations for this spectrum as 10-15 %. Fig. 4 illustrates the differential radiation energy losses for 10 GeV positrons which move in the (110) plane of 0.045 cm silicon single crystal with the 5 m bending radius.

Note that the photon spectrum should be averaged over transversal energies as it was described in [4]. However, for large enough radii of bending (in comparison with the characteristic radius [4]) the averaged spectrum is approximately the same as calculated for one transversal energy.

In calculations we consider the pure process in a planar electric field of a single crystal and hence we do not take into account the thermal fluctuations of the potential and multiple scattering by atoms. Thermal fluctuations are the cause of appearance of amorphous contribution in the photon spectrum (see, for example [3, 5]). One can expect that the intensity of this process is close to similar one in straight single crystals. It means that in the range 20-40 GeV radiation at volume reflection exceed the amorphous contribution more than 30-40 times.

The multiple scattering can change the probability of radiation if angle of the scattering is close or exceed the characteristic angle of radiation on the formation length. It is easy to estimate that for conditions in Fig. 2 the angle due to multiple scattering (on the formation length) is less than 1μrad for photons with energies more than 10 GeV and therefore multiple scattering does not influent significantly on the radiation process.

It should be noted that Eq. (7) does not take into account multiple processes of photon radiation. However, presented in this paper photon spectra may be used for calculation of these processes.

4   Conclusion

It should emphasized in conclusion volume reflection radiation is in tens times more intensive than radiation in amorphous material. This new kind of radiation favorably differs also from radiation at channeling [6], because there is no sharp dependence on beam angular spread and sign of charge of a particle. For these reasons volume reflection radiation has exclusive perspectives for innovation in electron accelerators for creation of powerful radiation sources. Another possible application of volume reflection radiation is high-energy hadron and electron identification in physic experiments due to strong dependence of it parameters from particle mass.

The described type of radiation can be observed in radiation experiments, which are started at IHEP U-70[7] and CERN SPS [2, 8].

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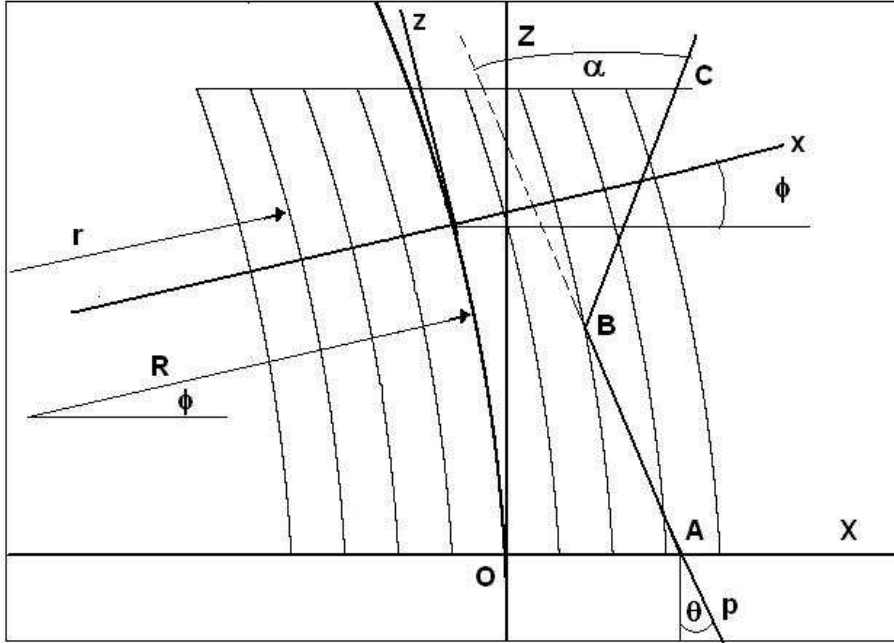
Figure 1: Scheme of the volume reflection of the proton beam. XYZ is the Cartesian coordinate system at the entrance in single crystal, xyz is the local Cartesian coordinate system connected with the current location of the particle. Y-axis is directed normally to the plane of figure. θ and α are the initial and volume reflection angles.
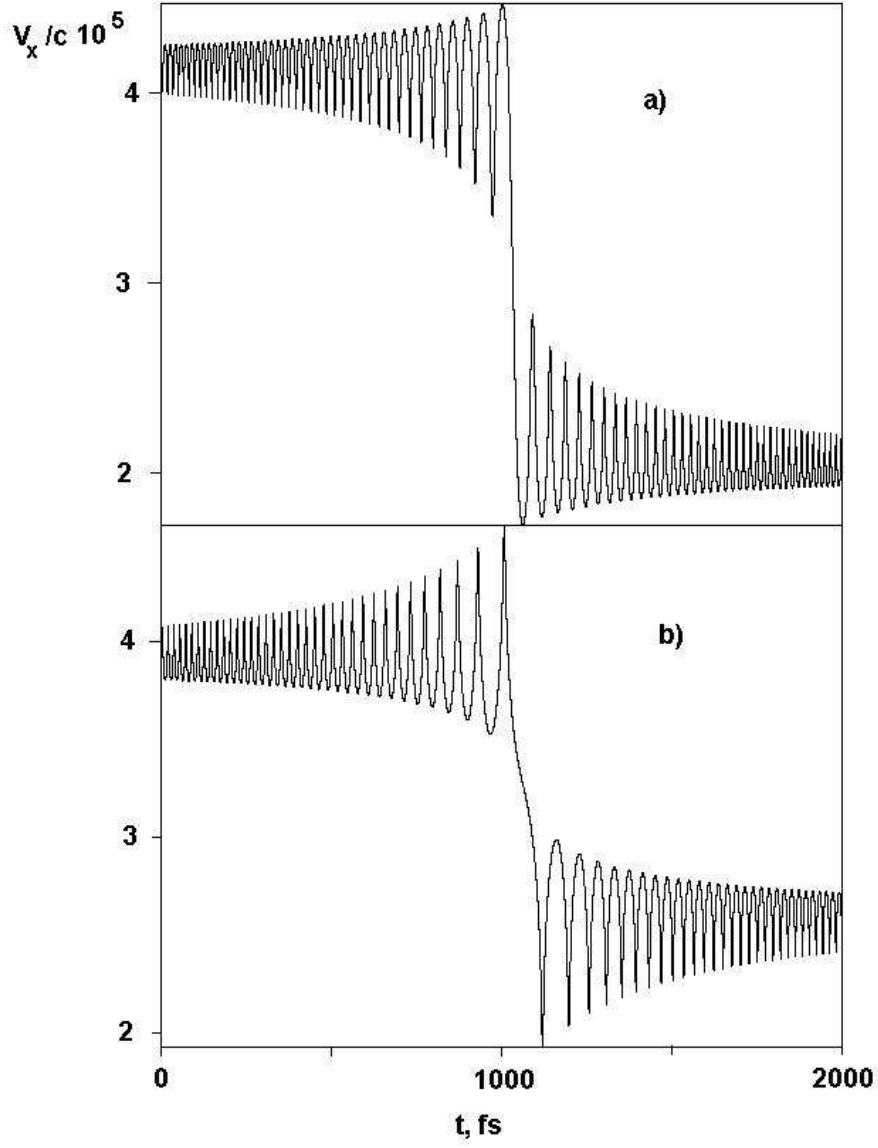
Figure 2: Relative transversal velocities of pozitrons (a) and electrons (b) at volume reflection in (110) plane of bent silicon single crystal as a functions of time. Particle energy is equal to 200 GeV, crystal thickness is 0.06 cm, radius of bending is 10 m.
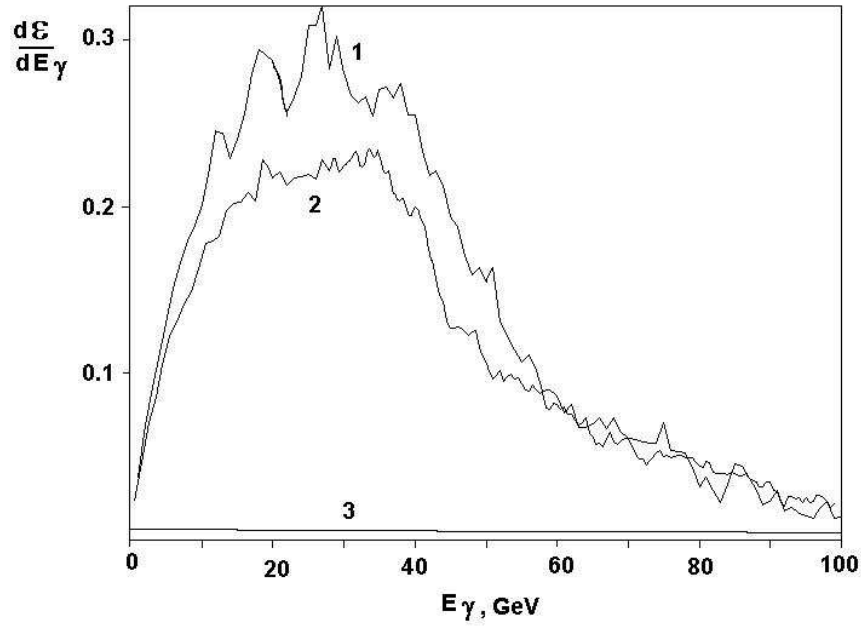
Figure 3: Differential radiation energy losses 200-GeV positrons (1) and electrons (2) for conditions as in Fig. 2. The curve 3 is amorphous contribution.
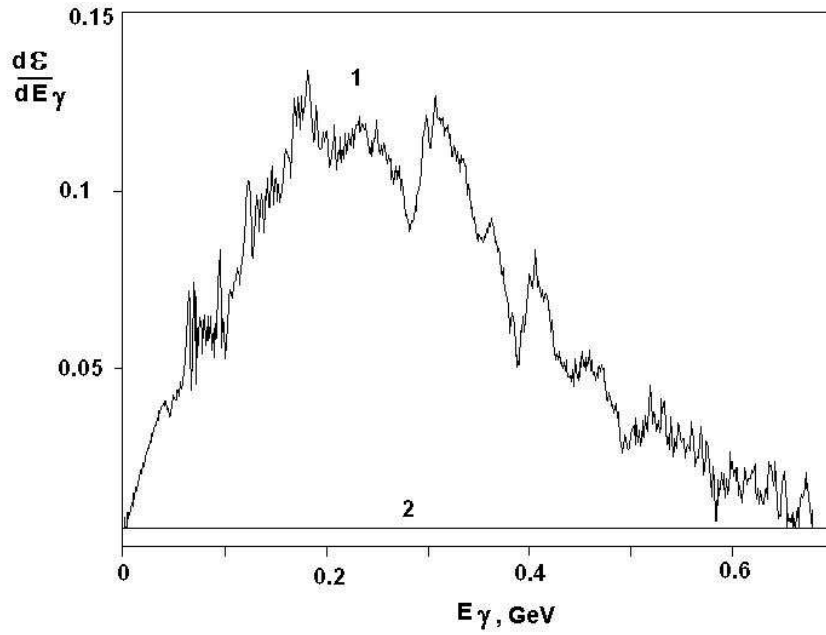

Figure 4: Differential radiation energy losses 10-GeV positrons (1) in 0.045 cm silicon single crystal. The curve 2 is amorphous contribution.